Well-posedness of interaction problems coupling a viscous fluid and an elastic structure

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Introduction

- elastic structure immersed in an incompressible viscous fluid
- fluid and structure contained in $\Omega \subset \mathbb{R}^3$ a fixed bounded and connected set
- fluid model: Navier-Stokes equations
- solid model: linearized elasticity equation
- coupling through conditions on the interface

Modelling

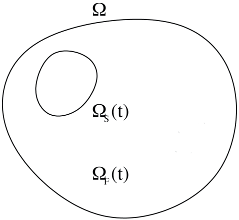
Fluid equations

$$\begin{cases} \partial_t u + (u \cdot \nabla) u - \nabla \cdot \mathbb{T}(u,p) = 0 \text{ in } \Omega_F(t) \\ \nabla \cdot u = 0 \text{ in } \Omega_F(t) \\ u = 0 \text{ on } \partial\Omega \\ u(0,\cdot) = u_0 \text{ in } \Omega_F \end{cases}$$

u: eulerian velocity, p pressure

$\mathbb{T}(u,p)$: Cauchy stress tensor given by

$$\mathbb{T}(u,p) = 2\mu\epsilon(u) - p\,\mathrm{Id}$$



Modelling

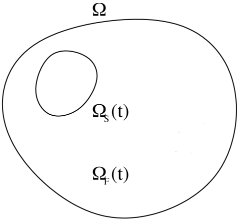
Solid equation

$$\begin{cases} \partial_{tt}\xi - \nabla \cdot \Sigma(\xi) = 0 \text{ in } \Omega_S \\ \xi(0,\cdot) = 0 \text{ in } \Omega_S,\ \partial_t \xi(0,\cdot) = \xi_1 \text{ in } \Omega_S. \end{cases}$$

ξ: elasticity displacement

$\Sigma(\xi)$: linear elasticity tensor

$$\Sigma(\xi) = 2\lambda\epsilon(\xi) + \lambda'(\nabla\cdot\xi)\mathrm{Id}$$



Modelling

Coupling conditions

$$\left\{ \begin{array}{rl} u \circ X & = \partial_t \xi \text{ on } \partial\Omega_S \\ \mathbb{T}(u,p) \circ X \,\mathrm{cof} \nabla X \,\mathrm{n} & = \Sigma(\xi)\,\mathrm{n} \text{ on } \partial\Omega_S \end{array} \right.$$

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Definition of the flow

for all $y \in \Omega_F$

$$\begin{cases} \partial_t X(t,y) & = u(t, X(t,y)), \quad t \in (0,T) \\ X(0,y) & = y \end{cases}$$

Modelling

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Remarks

- Eulerian point of view in the fluid versus lagrangian point of view in the structure.
- Fluid equations are given on a moving and unknown domain.

A priori energy estimate

$$\begin{cases} \partial_t u + (u \cdot \nabla) u - \nabla \cdot \mathbb{T}(u,p) = 0 & \text{in } \Omega_F(t) \\ \nabla \cdot u = 0 & \text{in } \Omega_F(t) \\ \partial_{tt}\xi - \nabla \cdot \Sigma(\xi) = 0 & \text{in } \Omega_S \\ u = 0 & \text{on } \partial\Omega \\ u \circ X = \partial_t \xi & \text{on } \partial\Omega_S \\ \mathbb{T}(u,p) \circ X \, \mathrm{cof} \nabla X \, \mathrm{n} = \Sigma(\xi) \, \mathrm{n} & \text{on } \partial\Omega_S \end{cases}$$

Energy spaces:

$u \in$ "$L^\infty(0,T;L^2(\Omega_F(t))) \cap L^2(0,T;H^1(\Omega_F(t)))$",

$\xi \in W^{1,\infty}(0,T;L^2(\Omega_S(0))) \cap L^\infty(0,T;H^1(\Omega_S(0)))$.

A priori energy estimate

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This regularity is insufficient:

- The set $\Omega_S(t) = (Id + \xi(t))(\Omega_S(0))$ is not Lipschitz.

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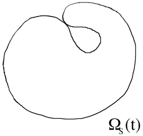
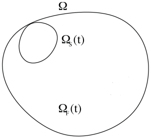
This regularity is insufficient:

- The set $\Omega_S(t) = (Id + \xi(t))(\Omega_S(0))$ is not Lipschitz.
- The flow in the structure domain $Id + \xi(t,\cdot)$ is a priori not invertible.
- We can instantaneously have self-contact, loss of orientation and collision with the boundary.



Self-contact



Collision

Remarks

Mismatch between parabolic and hyperbolic regularity

$$\begin{cases} \partial_t u - \Delta u = 0 & \text{in } (0,T) \times \Omega_F \\ \partial_{tt}\xi - \Delta \xi = 0 & \text{in } (0,T) \times \Omega_S \\ u = \partial_t \xi & \text{on } (0,T) \times \Sigma \\ \nabla u \cdot \mathrm{n} = \nabla \xi \cdot \mathrm{n} & \text{on } (0,T) \times \Sigma \end{cases}$$

Remarks

Mismatch between parabolic and hyperbolic regularity

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Energy-level space:

$$\xi \in L^\infty(H^1(\Omega_S))\cap W^{1,\infty}(L^2(\Omega_S)), \quad u \in L^\infty(L^2(\Omega_F))\cap L^2(H^1(\Omega_F))$$

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Remark: sense of the boundary conditions ?

[Barbu, Grujic, Lasiecka, Tuffaha (2007)]

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Hidden regularity for ξ:

$$\text{If } \partial_t\xi \text{ belongs to } L^2(H^{1/2}(\Sigma)), \text{ then } \nabla\xi\cdot\mathrm{n} \text{ belongs to } L^2(H^{-1/2}(\Sigma))$$

Bibliography

- Regularization of the elasticity equation: add of regularizing terms, finite number of modes

  [M.B. (2007)], [Desjardins, Esteban, Grandmont, Le Tallec (2001)], [M.B., Schwindt, Takahashi (2012)]

- Obtaining smooth solution

  [Coutand, Shkoller (2005, 2006)], [Kukavica, Tuffaha (2012)],
  [Raymond, Vanninathan (2014)], [M.B, Guerrero, Takahashi (2019)],
  [Ignatova, Kukavica, Lasiecka, Tuffaha (2014, 2017)]

Bibliography

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  [Ignatova, Kukavica, Lasiecka, Tuffaha (2014, 2017)]

- A different geometry: coupling 2D-1D or 3D-2D

  [Beirao da Veiga (2004)], [Guidorzi, Padula, Plotnikov (2008)],
  [Chambolle, Desjardins, Esteban, Grandmont (2005)], [Grandmont (2008)],
  [Muha, Canic (2014)], [Grandmont, Hillairet (2016)],
  [Galdi, Kyed (2018, 2020)], [Grandmont, Hillairet, Lequeurre (2019)],
  [Djebour, Takahashi (2019)]

Change of variables for the fluid equation

$$\begin{cases} \partial_t u + (u \cdot \nabla) u - \nabla \cdot \mathbb{T}(u,p) = 0 & \text{in } \Omega_F(t) \\ \nabla \cdot u = 0 & \text{in } \Omega_F(t) \\ u = 0 & \text{on } \partial\Omega \\ u \circ X = \partial_t \xi & \text{on } \partial\Omega_S \\ \mathbb{T}(u,p) \circ X \, \mathrm{cof} \nabla X \, \mathrm{n} = \Sigma(\xi) \, \mathrm{n} & \text{on } \partial\Omega_S \end{cases}$$

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We set in $(0,T) \times \Omega_F$

$$v(t,y) = u(t, X(t,y)), \quad q(t,y) = p(t, X(t,y))$$

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We set in $(0,T) \times \Omega_F$

$$v(t,y) = u(t, X(t,y)), \quad q(t,y) = p(t, X(t,y))$$

and we get

$$\begin{cases} \partial_t v - \nabla \cdot \mathbb{T}_X(v,q) = 0 & \text{in } \Omega_F \\ \nabla v : \mathrm{Cof}(\nabla X) = 0 & \text{in } \Omega_F \\ v = 0 & \text{on } \partial\Omega \\ v = \partial_t \xi & \text{on } \partial\Omega_S \\ \mathbb{T}_X(v,q)\, \mathrm{n} = \Sigma(\xi)\, \mathrm{n} & \text{on } \partial\Omega_S \end{cases}$$

where

$$\mathbb{T}_X(v,q) := [(\nabla v)\, \mathrm{Cof}(\nabla X)^* + \mathrm{Cof}(\nabla X)(\nabla v)^* - q\mathrm{Id}]\, \mathrm{Cof}(\nabla X).$$

Main result

M.B., S. Guerrero, T. Takahashi, *Nonlinearity (2019)*

Hypotheses:

- $d(\Omega_S, \partial\Omega) > 0$
- $u_0 \in H^2(\Omega_F)$, $\xi_1 \in H^{1+1/8}(\Omega_S)$
- compatibility conditions on the initial conditions

Theorem:

There exists a time $T > 0$ depending on $\|u_0\|_{H^2(\Omega_F)}$ and $\|\xi_1\|_{H^{9/8}(\Omega_S)}$ such that our system admits a unique solution (X, v, q, ξ) defined in $(0, T)$ in the following spaces:

$$v \in C^1(L^2(\Omega_F)) \cap H^1(H^1(\Omega_F)) \cap C^0(H^2(\Omega_F)) \cap L^2(H^{5/2+1/8}(\Omega_F))$$

$$q \in C^0(H^1(\Omega_F)) \cap L^2(H^{3/2+1/8}(\Omega_F))$$

$$\xi \in C^2(L^2(\Omega_S)) \cap C^1(H^{1+1/8}(\Omega_S)) \cap C^0(H^{2+1/8}(\Omega_S))$$

Moreover, $X(t, \cdot) : \Omega_F \to \Omega_F(t)$ is a diffeomorphism, for all $t \in (0, T)$.

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Moreover, $X(t,\cdot) : \Omega_F \to \Omega_F(t)$ is a diffeomorphism, for all $t \in (0, T)$.

Remarks:

- The regularity of the initial conditions is preserved over time.
- $\nabla X \in H^1(H^{3/2+\epsilon}(\Omega_F)) \hookrightarrow H^1(C^0(\overline{\Omega_F}))$ with $\epsilon = 1/8$: $\|\nabla X - \mathrm{Id}\|_{C^0([0,T]\times\overline{\Omega_F})} \leq CT^{1/2}$

Main result

M.B., S. Guerrero, T. Takahashi, *Nonlinearity (2019)*

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Moreover, $X(t, \cdot) : \Omega_F \to \Omega_F(t)$ is a diffeomorphism, for all $t \in (0, T)$.

Other results:

[Coutand, Shkoller (2005)]: more regular initial conditions

[Kukavica, Tuffaha (2012)], [Raymond, Vanninathan (2014)]: periodic boundary conditions, flat initial interface.

Linearization

We take $\widehat{X}$ in the set

$$B_M = \Big\{ \widehat{X} \in \mathcal{X}_T := C^2(L^2(\Omega_F)) \cap H^2(H^1(\Omega_F)) \cap C^1(H^2(\Omega_F)) \cap H^1(H^{5/2+1/8}(\Omega_F)) \\ \|\widehat{X}\|_{\mathcal{X}_T} \leq M, \widehat{X}(0,\cdot) = \mathrm{Id} \text{ and } \partial_t \widehat{X}(0,\cdot) = u_0 \text{ in } \Omega_F \Big\}$$

$$\begin{cases} \partial_t v - \nabla \cdot \mathbb{T}_X(v,q) = 0 & \text{in } (0,T) \times \Omega_F \\ \nabla v : \mathrm{Cof}(\nabla X) = 0 & \text{in } (0,T) \times \Omega_F \\ \partial_{tt}\xi - \nabla \cdot \Sigma(\xi) = 0 & \text{in } (0,T) \times \Omega_S \\ v = 0 & \text{on } (0,T) \times \partial\Omega \\ v = \partial_t \xi & \text{on } (0,T) \times \partial\Omega_S \\ \mathbb{T}_X(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} & \text{on } (0,T) \times \partial\Omega_S \\ v(0,\cdot) = u_0 \text{ in } \Omega_F,\ \xi(0,\cdot) = 0 & \text{in } \Omega_S, \partial_t\xi(0,\cdot) = \xi_1 \text{ in } \Omega_S. \end{cases}$$

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$$\begin{cases} \partial_t v - \nabla \cdot \mathbb{T}_{\widehat{X}}(v,q) = 0 & \text{in } (0,T)\times\Omega_F \\ \nabla v : \mathrm{Cof}(\nabla \widehat{X}) = 0 & \text{in } (0,T)\times\Omega_F \\ \partial_{tt}\xi - \nabla\cdot\Sigma(\xi) = 0 & \text{in } (0,T)\times\Omega_S \\ v = 0 & \text{on } (0,T)\times\partial\Omega \\ v = \partial_t \xi & \text{on } (0,T)\times\partial\Omega_S \\ \mathbb{T}_{\widehat{X}}(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} & \text{on } (0,T)\times\partial\Omega_S \\ v(0,\cdot) = u_0 \text{ in } \Omega_F,\ \xi(0,\cdot) = 0 & \text{in } \Omega_S, \partial_t\xi(0,\cdot) = \xi_1 \text{ in } \Omega_S. \end{cases}$$

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- Existence and uniqueness of solution for this system.
- Fixed point for the map $\Lambda : \widehat{X} \in B_M \to X \in B_M$ where $X(t,\cdot) = \mathrm{Id} + \displaystyle\int_0^t v(s,\cdot)\, ds$.

Two subproblems

We consider the following two subproblems:

$$\begin{cases} \partial_{tt}\xi - \nabla \cdot \Sigma(\xi) = 0 & \text{in } (0, T) \times \Omega_S \\ \xi(t, \cdot) = \displaystyle\int_0^t v(s, \cdot)\, ds & \text{on } (0, T) \times \partial\Omega_S \\ \xi(0, \cdot) = 0, \ \partial_t\xi(0, \cdot) = \xi_1 & \text{in } \Omega_S \end{cases}$$

$$\begin{cases} \partial_t v - \nabla \cdot \mathbb{T}_{\widehat{X}}(v, q) = 0 & \text{in } (0, T) \times \Omega_F \\ \nabla v : \mathrm{Cof}(\nabla \widehat{X}) = 0 & \text{in } (0, T) \times \Omega_F \\ v = 0 & \text{on } (0, T) \times \partial\Omega \\ \mathbb{T}_{\widehat{X}}(v, q)\, \mathrm{n} = \Sigma(\xi)\, \mathrm{n} & \text{on } (0, T) \times \partial\Omega_S \\ v(0, \cdot) = u_0 & \text{in } \Omega_F\,. \end{cases}$$

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$$\begin{cases} \partial_t v - \nabla\cdot\mathbb{T}_{\widehat{X}}(v,q) = 0 & \text{in } (0,T)\times\Omega_F \\ \nabla v : \mathrm{Cof}(\nabla\widehat{X}) = 0 & \text{in } (0,T)\times\Omega_F \\ v = 0 & \text{on } (0,T)\times\partial\Omega \\ \mathbb{T}_{\widehat{X}}(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} & \text{on } (0,T)\times\partial\Omega_S \\ v(0,\cdot) = u_0 & \text{in } \Omega_F. \end{cases}$$

We take $(\tilde{v},\tilde{q})$ and we introduce two new subproblems:

$$\begin{cases} \partial_{tt}\xi - \nabla\cdot\Sigma(\xi) = 0 & \text{in } (0,T)\times\Omega_S \\ \xi(t,\cdot) = \displaystyle\int_0^t \tilde{v}(s,\cdot)\,ds & \text{on } (0,T)\times\partial\Omega_S \\ \xi(0,\cdot)=0,\ \partial_t\xi(0,\cdot)=\xi_1 & \text{in } \Omega_S \end{cases}$$

$$\begin{cases} \partial_t v - \nabla\cdot\mathbb{T}(v,q) = F_1(\tilde{v},\tilde{q}) & \text{in } (0,T)\times\Omega_F \\ \nabla\cdot v = F_2(\tilde{v},\tilde{q}) & \text{in } (0,T)\times\Omega_F \\ v = 0 & \text{on } (0,T)\times\partial\Omega \\ \mathbb{T}(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} + F_3(\tilde{v},\tilde{q}) & \text{on } (0,T)\times\partial\Omega_S \\ v(0,\cdot) = u_0 & \text{in } \Omega_F. \end{cases}$$

with

$$F_1(\tilde{v},\tilde{q}) = \nabla\cdot(\mathbb{T}_{\widehat{X}}(\tilde{v},\tilde{q}) - \mathbb{T}(\tilde{v},\tilde{q})),\ F_2(\tilde{v},\tilde{q}) = \nabla v : (\mathrm{Id} - \mathrm{Cof}(\nabla\widehat{X})),\ F_3(\tilde{v},\tilde{q}) = (\mathbb{T}(\tilde{v},\tilde{q}) - \mathbb{T}_{\widehat{X}}(\tilde{v},\tilde{q}))\,\mathrm{n}$$

Two subproblems

We consider the following two subproblems:

$$\begin{cases} \partial_{tt}\xi - \nabla\cdot\Sigma(\xi) = 0 & \text{in } (0,T)\times\Omega_S \\ \xi(t,\cdot) = \displaystyle\int_0^t v(s,\cdot)\,ds & \text{on } (0,T)\times\partial\Omega_S \\ \xi(0,\cdot)=0,\ \partial_t\xi(0,\cdot)=\xi_1 & \text{in } \Omega_S \end{cases}$$

$$\begin{cases} \partial_t v - \nabla\cdot\mathbb{T}_{\widehat{X}}(v,q) = 0 & \text{in } (0,T)\times\Omega_F \\ \nabla v : \mathrm{Cof}(\nabla\widehat{X}) = 0 & \text{in } (0,T)\times\Omega_F \\ v = 0 & \text{on } (0,T)\times\partial\Omega \\ \mathbb{T}_{\widehat{X}}(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} & \text{on } (0,T)\times\partial\Omega_S \\ v(0,\cdot) = u_0 & \text{in } \Omega_F. \end{cases}$$

We take $(\tilde v,\tilde q)$ and we introduce two new subproblems:

$$\begin{cases} \partial_{tt}\xi - \nabla\cdot\Sigma(\xi) = 0 & \text{in } (0,T)\times\Omega_S \\ \xi(t,\cdot) = \displaystyle\int_0^t \tilde v(s,\cdot)\,ds & \text{on } (0,T)\times\partial\Omega_S \\ \xi(0,\cdot)=0,\ \partial_t\xi(0,\cdot)=\xi_1 & \text{in } \Omega_S \end{cases}$$

$$\begin{cases} \partial_t v - \nabla\cdot\mathbb{T}(v,q) = F_1(\tilde v,\tilde q) & \text{in } (0,T)\times\Omega_F \\ \nabla\cdot v = F_2(\tilde v,\tilde q) & \text{in } (0,T)\times\Omega_F \\ v = 0 & \text{on } (0,T)\times\partial\Omega \\ \mathbb{T}(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} + F_3(\tilde v,\tilde q) & \text{on } (0,T)\times\partial\Omega_S \\ v(0,\cdot) = u_0 & \text{in } \Omega_F. \end{cases}$$

with

$$F_1(\tilde v,\tilde q) = \nabla\cdot(\mathbb{T}_{\widehat{X}}(\tilde v,\tilde q) - \mathbb{T}(\tilde v,\tilde q)),\ F_2(\tilde v,\tilde q) = \nabla v : (\mathrm{Id} - \mathrm{Cof}(\nabla\widehat{X})),\ F_3(\tilde v,\tilde q) = (\mathbb{T}(\tilde v,\tilde q) - \mathbb{T}_{\widehat{X}}(\tilde v,\tilde q))\,\mathrm{n}$$

Fixed point for the map

$$(\tilde v,\tilde q)\in\mathcal{S}_1\times\mathcal{S}_2 \to (v,q)\in\mathcal{S}_1\times\mathcal{S}_2$$

Very regular solution for the linear system with smoother initial conditions

We define

$$\mathcal{S}_1 = H^2(L^2(\Omega_F)) \cap H^1(H^2(\Omega_F)) \cap L^2(H^{5/2+1/8}(\Omega_F))$$

and

$$\mathcal{S}_2 = H^1(H^1(\Omega_F)) \cap L^2(H^{3/2+1/8}(\Omega_F))$$

and we assume that $u_0 \in H^3(\Omega_F)$ and $\xi_1 \in H^{3/2+1/8}(\Omega_S)$ (+ compatibility conditions).

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Main theorem:

$$v \in C^1(L^2(\Omega_F)) \cap C^0(H^2(\Omega_F)) \cap L^2(H^{5/2+1/8}(\Omega_F))$$

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$$u_0 \in H^2(\Omega_F), \quad \xi_1 \in H^{1+1/8}(\Omega_S)$$

Two steps:

- If $(\tilde{v}, \tilde{q}) \in \mathcal{S}_1 \times \mathcal{S}_2$, then $(v, q) \in \mathcal{S}_1 \times \mathcal{S}_2$.
- $(\tilde{v}, \tilde{q}) \to (v, q)$ is a contraction: $\|(v, q)\|_{\mathcal{S}_1 \times \mathcal{S}_2} \leq CT^\alpha M \|(\tilde{v}, \tilde{q})\|_{\mathcal{S}_1 \times \mathcal{S}_2}$ when $(u_0, \xi_1) = (0, 0)$.

Regularity result for the elasticity equation: hidden regularity results

[Lions, Lasiecka, Triggiani (1986)] [Raymond, Vanninathan (2014)], [Dehman, Raymond (2015)]

Let w be a solution of

$$\left\{ \begin{aligned} \partial_{tt} w - \Delta w = 0 & \quad \text{in } (0,T)\times\Omega_S \\ w = f & \quad \text{on } (0,T)\times\partial\Omega_S \\ w(0) = w_0,\, \partial_t w(0) = w_1 & \quad \text{in } \Omega_S. \end{aligned} \right.$$

We assume that

$$w_0 \in H^1(\Omega_S) \text{ and } w_1 \in L^2(\Omega_S)$$

and

$$f \in H^1((0,T)\times\partial\Omega_S).$$

Then, there exists a solution w such that

$$w \in C([0,T]; H^1(\Omega_S)) \cap C^1([0,T]; L^2(\Omega_S))$$

And the normal derivative of w satisfies

$$\nabla w\, n \in L^2((0,T)\times\partial\Omega_S).$$

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Let $\alpha \in [0,2]$. We assume that

$$w_0 \in H^\alpha(\Omega_S) \text{ and } w_1 \in H^{\alpha-1}(\Omega_S)$$

and

$$f \in H^\alpha((0,T) \times \partial\Omega_S).$$

Then, there exists a solution w such that

$$w \in C([0,T]; H^\alpha(\Omega_S)) \cap C^1([0,T]; H^{\alpha-1}(\Omega_S))$$

And the normal derivative of w satisfies

$$\nabla w\, n \in H^{\alpha-1}((0,T) \times \partial\Omega_S).$$

Regularity results for the subproblems

Hidden regularity result [Lions, Lasiecka, Triggiani (1986)], [Raymond, Vanninathan (2014)], [Dehman, Raymond (2015)]

$$\begin{cases} \partial_{tt}(\partial_t \xi) - \nabla \cdot \Sigma(\partial_t \xi) = 0 & \text{in } (0,T) \times \Omega_S \\ \partial_t \xi = \tilde{v} & \text{on } (0,T) \times \partial\Omega_S \\ \partial_t \xi(0,\cdot) = 0, \ \partial_{tt}\xi(0,\cdot) = 0 & \text{in } \Omega_S \end{cases}$$

We have $\tilde{v} \in H^{3/2+1/8}((0,T) \times \partial\Omega_S)$. We deduce that

$$\|\partial_t \xi\|_{C^0(H^{3/2+1/8}(\Omega_S)) \cap C^1(H^{1/2+1/8}(\Omega_S))} + \|\Sigma(\partial_t \xi) n\|_{H^{1/2+1/8}((0,T)\times\partial\Omega_S)} \leq C T^{\alpha} \|\tilde{v}\|_{\mathcal{S}_1}.$$

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We have $\tilde{v}\in H^{3/2+1/8}((0,T)\times\partial\Omega_S)$. We deduce that

$$\|\partial_t\xi\|_{C^0(H^{3/2+1/8}(\Omega_S))\cap C^1(H^{1/2+1/8}(\Omega_S))} + \|\Sigma(\partial_t\xi)n\|_{H^{1/2+1/8}((0,T)\times\partial\Omega_S)} \le CT^\alpha\|\tilde{v}\|_{\mathcal{S}_1}.$$

Regularity result for Stokes problem [Grubb, Solonnikov (1991)]

$$\begin{cases} \partial_t(\partial_t v) - \nabla\cdot\mathbb{T}(\partial_t v,\partial_t q) = \partial_t F_1(\tilde{v},\tilde{q}) & \text{in } (0,T)\times\Omega_F \\ \nabla\cdot(\partial_t v) = \partial_t F_2(\tilde{v},\tilde{q}) & \text{in } (0,T)\times\Omega_F \\ \partial_t v = 0 & \text{on } (0,T)\times\partial\Omega \\ \mathbb{T}(\partial_t v,\partial_t q)\,\mathrm{n} = \Sigma(\partial_t\xi)\,\mathrm{n} + \partial_t F_3(\tilde{v},\tilde{q}) & \text{on } (0,T)\times\partial\Omega_S \\ \partial_t v(0,\cdot) = 0 & \text{in } \Omega_F. \end{cases}$$

We deduce that

$$\|\partial_t v\|_{L^2(H^2(\Omega_F))\cap H^1(L^2(\Omega_F))} + \|\partial_t q\|_{L^2(H^1(\Omega_F))} \le C(\|\Sigma(\partial_t\xi)\,\mathrm{n}\|_{H^{1/2}((0,T)\times\partial\Omega_S)} + T^\alpha\|(\tilde{v},\tilde{q})\|_{\mathcal{S}_1\times\mathcal{S}_2})$$

Regularity results for the subproblems

Thus, we have

$$\|\xi\|_{C^1(H^{3/2+1/8}(\Omega_S))\cap C^2(H^{1/2+1/8}(\Omega_S))} + \|v\|_{H^1(H^2(\Omega_F))\cap H^2(L^2(\Omega_F))} + \|q\|_{H^1(H^1(\Omega_F))} \leq CT^{\alpha}\|(\tilde{v},\tilde{q})\|_{\mathcal{S}_1\times\mathcal{S}_2}.$$

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It remains to get more spatial regularity:

$\xi \in C^0(H^{5/2+1/8}(\Omega_S))$, $v \in L^2(H^{5/2+1/8}(\Omega_F))$ and $q \in L^2(H^{3/2+1/8}(\Omega_F))$:

Regularity results for the subproblems

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$$\begin{cases} -\nabla\cdot\Sigma(\xi) = -\partial_{tt}\xi & \text{in } (0,T)\times\Omega_S \\ \xi(t,\cdot) = \displaystyle\int_0^t \tilde{v}(s,\cdot)\,ds & \text{on } (0,T)\times\partial\Omega_S \end{cases}$$

We get:

$$\|\xi\|_{C^0(H^{5/2+1/8}(\Omega_S))} \leq C\big(T^{\alpha}\|(\tilde{v},\tilde{q})\|_{\mathcal{S}_1\times\mathcal{S}_2} + T^{1/2}\|\tilde{v}\|_{L^2(H^{5/2+1/8}(\Omega_F))}\big)$$

Regularity results for the subproblems

Thus, we have

$$\|\xi\|_{C^1(H^{3/2+1/8}(\Omega_S))\cap C^2(H^{1/2+1/8}(\Omega_S))} + \|v\|_{H^1(H^2(\Omega_F))\cap H^2(L^2(\Omega_F))} + \|q\|_{H^1(H^1(\Omega_F))} \leq CT^{\alpha}\|(\tilde{v},\tilde{q})\|_{\mathcal{S}_1\times\mathcal{S}_2}.$$

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We get:

$$\|\xi\|_{C^0(H^{5/2+1/8}(\Omega_S))} \leq C(T^{\alpha}\|(\tilde{v},\tilde{q})\|_{\mathcal{S}_1\times\mathcal{S}_2} + T^{1/2}\|\tilde{v}\|_{L^2(H^{5/2+1/8}(\Omega_F))})$$

$$\begin{cases} -\nabla\cdot\mathbb{T}(v,q) = -\partial_t v + F_1(\tilde{v},\tilde{q}) & \text{in } (0,T)\times\Omega_F \\ \nabla\cdot v = F_2(\tilde{v},\tilde{q}) & \text{in } (0,T)\times\Omega_F \\ v = 0 & \text{on } (0,T)\times\partial\Omega \\ \mathbb{T}(v,q)\,\mathrm{n} = \Sigma(\xi)\,\mathrm{n} + F_3(\tilde{v},\tilde{q}) & \text{on } (0,T)\times\partial\Omega_S. \end{cases}$$

We have:

$$\|v\|_{L^2(H^{5/2+1/8}(\Omega_F))} + \|q\|_{L^2(H^{3/2+1/8}(\Omega_F))} \leq C(T^{\alpha}\|(\tilde{v},\tilde{q})\|_{\mathcal{S}_1\times\mathcal{S}_2} + \|\Sigma(\xi)\,\mathrm{n}\|_{L^2(H^{1+1/8}(\partial\Omega_S))})$$

Very regular solution for the linear system with smoother initial conditions

Proposition:

We assume that $u_0 \in H^3(\Omega_F)$ and $\xi_1 \in H^{3/2+1/8}(\Omega_S)$ (+ compatibility conditions).
There exists a time $T > 0$ depending on M such that the linear problem admits a unique solution:

$$v \in H^2(L^2(\Omega_F)) \cap H^1(H^2(\Omega_F)) \cap L^2(H^{5/2+1/8}(\Omega_F)) := \mathcal{S}_1$$

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Main theorem:

$$u_0 \in H^2(\Omega_F), \quad \xi_1 \in H^{1+1/8}(\Omega_S)$$

$$v \in \mathcal{R}_1 = C^1(L^2(\Omega_F)) \cap H^1(H^1(\Omega_F)) \cap C^0(H^2(\Omega_F)) \cap L^2(H^{5/2+1/8}(\Omega_F))$$

$$q \in \mathcal{R}_2 = C^0(H^1(\Omega_F)) \cap L^2(H^{3/2+1/8}(\Omega_F))$$

$$\xi \in C^2(L^2(\Omega_S)) \cap C^1(H^{1+1/8}(\Omega_S)) \cap C^0(H^{2+1/8}(\Omega_S))$$

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$$q \in \mathcal{R}_2 = C^0(H^1(\Omega_F)) \cap L^2(H^{3/2+1/8}(\Omega_F))$$

$$\xi \in C^2(L^2(\Omega_S)) \cap C^1(H^{1+1/8}(\Omega_S)) \cap C^0(H^{2+1/8}(\Omega_S))$$

We prove that the solution (v, q) satisfies estimates in $\mathcal{R}_1 \times \mathcal{R}_2$ of the form

$$\|(v,q)\|_{\mathcal{R}_1 \times \mathcal{R}_2} \leq C(\|u_0\|_{H^2(\Omega_F)} + \|\xi_1\|_{H^{1+1/8}})$$

in order to relax the regularity of the initial conditions.

More general initial conditions

We prove that this very regular solution satisfies

$$\|v\|_{C^1(L^2(\Omega_F))\cap H^1(H^1(\Omega_F))} + \|v\|_{C^0(H^2(\Omega_F))\cap L^2(H^{5/2+1/8}(\Omega_F))} \\ + \|\xi\|_{C^2(L^2(\Omega_S))\cap C^1(H^{1+1/8}(\Omega_S))} + \|\xi\|_{C^0(H^{2+1/8}(\Omega_S))} \leq C(\|u_0\|_{H^2(\Omega_F)} + \|\xi_1\|_{H^{1+1/8}(\Omega_S)})$$

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Arguments:

- energy estimate satisfied by (v, ξ),
- energy estimate satisfied by $(\partial_t v, \partial_t \xi)$,

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$$+ \|\xi\|_{C^2(L^2(\Omega_S))\cap C^1(H^{1+1/8}(\Omega_S))} + \|\xi\|_{C^0(H^{2+1/8}(\Omega_S))} \leq C(\|u_0\|_{H^2(\Omega_F)} + \|\xi_1\|_{H^{1+1/8}(\Omega_S)})$$

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in order to relax the regularity of the initial conditions.

Arguments:

- energy estimate satisfied by (v, ξ),
- energy estimate satisfied by $(\partial_t v, \partial_t \xi)$,
- elliptic estimates
- more spatial regularity obtained thanks to
  - hidden regularity results [Raymond, Vanninathan (2014)]
  - regularity results for Stokes system.

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  - regularity results for Stokes system.

At last, we use a fixed point argument to conclude.

Concluding remarks

- Global in time smooth solution with small data ?
  Incompressible fluid and damped wave equation
  [Ignatova, Kukavica, Lasiecka, Tuffaha (2014, 2017)]
- Extension to the nonlinear elasticity equation ?
  [Coutand, Shkoller (2006)], [M.B., Guerrero (2017)]